Module 04
Block Diagrams and Graphical Representations of Intertwined Dynamic Systems

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Module 4 Outline

1. Introduction to block diagrams
2. Physical meaning and importance
3. Block diagram reduction
4. Examples
   - Reading material: Dorf & Bishop, Section 2.6
Examples of Block Diagrams — Op Amps (Eww)
Examples of Block Diagrams — Temperature Control

Introduction to Block Diagrams
Basics of Block Diagrams
Examples of Block Diagram Simplifications

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Importance of Block Diagrams

- Graphical representation of interconnected systems are important.
- A system may consist of multiple subsystems: the output of one may be the input to another, and so on.
- Each subsystem is represented by a functional block, labeled with the corresponding transfer function.
- Blocks are connected by arrows to indicate signal flow directions.

**Advantages:**
- Easy for visualization purpose.
- Can represent a class of similar systems.
- Most importantly: can infer overall relationship between inputs and outputs, and hence analyze the system stability and performance.
Objective of Block Diagram Representation

- **Main objective:** reduce intertwined blocks of subsystems into one unified block or 1 TF

- **Implications:** given an overall TF for a system of subsystems, we can compactly analyze the dynamics of the system via one equation that depicts the dynamics.

- **Example:**

  ![Block Diagram Example](image-url)
Cascaded/Parallel Connected Systems

Cascaded systems:

\[ U(s) \xrightarrow{G_1(s)} Z(s) \xrightarrow{G_2(s)} Y(s) \]

Parallel connected systems:

\[ U(s) \xrightarrow{G_1(s)} Y_1(s) + Y_2(s) \xrightarrow{G_2(s)} Y(s) \]
In this class, we will be studying how to design the above system.

The above block representation is so common for so many systems.

**Control (reference) input:** $U(s)$ ($R(s)$); **Output:** $Y(s)$

**Plant dynamics:** $G(s)$—this is often given, defines physical systems.

**Control objective:** design $H(s)$ (gain) so that the system is stable.

$H(s), G(s)$ are all internal transfer functions, mapping their inputs to defined outputs.
(Negative) Feedback System\(^1\) — 1

- Negative feedback occurs when some function of the output is fed back to reduce the output fluctuations.
- Fluctuations often caused by changes in the input or disturbances.
- *Well, why not +ve feedback? Who likes −ve feedback anyway?*
- Hmm, in control systems the theme is different.
- If someone’s applying -ve feedback, then they’re most likely helping you.
- +ve feedback tends to lead to instability via exponential growth.
- −ve feedback promotes stability and error minimization.
- −ve feedback applications: electrical & mechanical systems, economics, nature, chemistry.

\(^1\)From Wikipedia...Oh and don’t ever let anyone lecture you when you get your very *basic* research from Wikipedia.
(Negative) Feedback System — 2

- **Tracking error**: \( E(s) = R(s) - B(s) \)

- **Feedforward transfer function (FTF)**: \( \frac{Y(s)}{E(s)} = G(s) \)

- **Open-loop transfer function (OLTF)**: \( \frac{B(s)}{E(s)} = G(s)H(s) \)

- **Closed-loop transfer function (CLTF)**: \( \frac{Y(s)}{R(s)} = ?? \)
Examples

- Example 1: *What if we have positive feedback?*
  
  **Solution:**

- Example 2: *What if \( H(s) = 1 \)? Unity feedback?*
  
  **Solution:**
Example 3: What is the CLTF for the above system?

By adjusting the controller $C(s)$, one can change the CLTF to achieve desired properties.

This control architecture is different than the one previously discussed.

However, both can provide desired system performance.
Objective: find the CLTF, \( \frac{Y(s)}{U(s)} \)

Solution:

The above example is simple, but sometimes things can be messy

Hence, we need block diagram transformations
### Important Block Diagram Transformations — 1

#### Block Diagram Transformations [Taken from Dorf & Bishop Textbook]

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Original Diagram</th>
<th>Equivalent Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Combining blocks in cascade</td>
<td>$X_1 \xrightarrow{G_1(s)} X_2 \xrightarrow{G_2(s)} X_3$</td>
<td>$X_1 \xrightarrow{G_1G_2} X_3$ or $X_1 \xrightarrow{G_2G_1} X_3$</td>
</tr>
<tr>
<td>2. Moving a summing point behind a block</td>
<td>$X_1 \xrightarrow{+} \xrightarrow{G} X_3$</td>
<td>$X_1 \xrightarrow{G} \xrightarrow{+} X_3$</td>
</tr>
<tr>
<td>3. Moving a pickoff point ahead of a block</td>
<td>$X_1 \xrightarrow{G} X_2$</td>
<td>$X_1 \xrightarrow{G} X_2$</td>
</tr>
</tbody>
</table>
Important Block Diagram Transformations — 2

Block Diagram Transformations [Taken from Dorf & Bishop Textbook]

<table>
<thead>
<tr>
<th>Transformation</th>
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<th>Equivalent Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. Moving a pickoff point behind a block</td>
<td><img src="image1" alt="Original Diagram" /></td>
<td><img src="image2" alt="Equivalent Diagram" /></td>
</tr>
<tr>
<td>5. Moving a summing point ahead of a block</td>
<td><img src="image3" alt="Original Diagram" /></td>
<td><img src="image4" alt="Equivalent Diagram" /></td>
</tr>
<tr>
<td>6. Eliminating a feedback loop</td>
<td><img src="image5" alt="Original Diagram" /></td>
<td><img src="image6" alt="Equivalent Diagram" /></td>
</tr>
</tbody>
</table>
Block Diagram Simplification

- Find the CLTF utilizing the previous transformations
- Hint: use property 4 (see previous slide)
- Property 4: sliding a branch point past a function block
Solution to the Previous Example
Another Approach

Can we use another property?

Yes, we can use Property 5 (moving a summing point ahead of a block)
Solution via Property 5
Another Example

- **Solution:** First, let’s move $H_2$ behind block $G_4$ so that we can isolate the $G_3 - G_4 - H_1$ feedback loop.

- Again, we use Property 4 to get:
Solution to the Previous Example

Given the block diagram:

\[ R \quad + \quad G_1 \quad + \quad G_2 \quad \frac{G_3 \cdot G_4}{1 - G_3 \cdot G_4 \cdot H_1} \quad H_3 \quad \frac{G_2 \cdot G_3 \cdot G_4}{1 - G_3 \cdot G_4 \cdot H_1 + G_2 \cdot G_3 \cdot H_2} \quad H_3 \quad G_1 \cdot G_2 \cdot G_3 \cdot G_4 \quad 1 - G_3 \cdot G_4 \cdot H_1 + G_2 \cdot G_3 \cdot H_2 + G_1 \cdot G_2 \cdot G_3 \cdot G_4 \cdot H_3 \quad Y \]
The previous approach can be a bit tricky in some scenarios

It’s a great approach if you can see things easily

If you can’t or don’t want to, there’s a more algorithmic approach

Mason’s Formula:

– A systematic way to compute TFs from any input to any output

– Based on an algorithmic method and signal flow graphs

– Not discussed in class, but you can read more about (Mason’s Gain Rule handout on Blackboard)
Roadmap Revisited

Modeling (5-6 Weeks)
- Laplace Transforms
- Transfer Functions
- Solution of ODEs
- Modeling of Systems
- Block Diagrams
- Linearization

Analysis (7-8 Weeks)
- 1st & 2nd Order Systems
  - Time Response
  - Transient & Steady State
- Frequency Response
- Bode Plots
- RH Criterion
- Stability Analysis

Design (5-6 Weeks)
- Root-Locus
- Modern Control
- State-Space
- MIMO System Properties
Questions And Suggestions?

Any questions?

Thank You!

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IFF you want to know more 😊