Given the following plant dynamics:
\[
\dot{x}_p = A_p x_p + B_p^{(1)} u_1 + B_p^{(2)} u_2 \\
y = C_p x_p, \quad x_p(0) \text{ not given}
\]
where \(u_2(t)\) is the unknown input vector. The system consists of \(n\) states, \(m_1\) known inputs, \(m_2\) unknown inputs, and \(p\) measurable outputs. We want to design a dynamic unknown input observer (UIO) which takes the following form:
\[
\dot{x}_c = A_c x_c + B_c^{(1)} y + B_c^{(2)} u_1, \\
\hat{x}_p = x_c + My,
\]
The UIO is motivated by writing \(x_p\) as:
\[
x_p = (I - MC_p)x_p + MC_p x_p = x_c + My.
\]
1. Assume that the updated \(x_c\) takes the following form:
\[
x_c = (I - MC_p)x_p.
\]
   (a) Find \(\dot{x}_c = A_c x_c + B_c^{(1)} y + B_c^{(2)} u_1\), where \(A_c, B_c^{(1)}, B_c^{(2)}\) are matrices that you should determine, assuming that the unknown input vector is nullified and a convergence term is added to \(x_c\), as discussed in class. Note that \(\hat{x}_p = x_c + My\);
   (b) Derive the matrix equality that guarantees the nullification of \(u_2(t)\).

Precisely, you should find \(A_c, B_c^{(1)}, B_c^{(2)}\) in terms of \(A_p, B_p^{(1)}, C_p, M, L\).
2. If $p = m = n$, and $C_p B_p^{(2)}$ is invertible (and obviously square), what is a closed-form solution for the design matrix $M$?

3. Derive the estimation error dynamics.