A dynamical CTLTI system is characterized by $A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}, C = [0.5 \ 1]$. 

1. Find a linear state-observer gain $L = [l_1 \ l_2]^T$ such that the poles of the estimation error are $-5$ and $-7$.

2. Can you place both poles at $-6$? If yes, what is the corresponding observer gain?

Solutions:

1. First, we find $A - LC$ in terms of $l_1$ and $l_2$:

$$A - LC = \begin{bmatrix} 1 - l_1/2 & 3 - l_1 \\ 3 - l_2/2 & 1 - l_2 \end{bmatrix}.$$ 

Since the roots of the designed observer are $-5$ and $-7$, the desired characteristic polynomial is:

$$\pi_{A-LC} = (\lambda + 5)(\lambda + 7) = \lambda^2 + 12\lambda + 35.$$ 

The characteristic polynomial in terms of $l_1$ and $l_2$ can be written as:

$$+\lambda^2 + \lambda \left(-2 + \frac{l_1}{2} + l_2\right) -8 \left(\frac{5l_1}{2} + \frac{l_2}{2}\right) = 0.$$ 

Solving the following linear system of equations,

$$35 = -8 + \frac{5l_1}{2} + \frac{l_2}{2}$$

$$12 = -2 + \frac{l_1}{2} + l_2,$$

we obtain $l_1 = 16$ and $l_2 = 6$.

2. Placing poles at $\lambda = -6$ means that

$$\pi_{A-LC} = (\lambda + 6)^2 = \lambda^2 + 12\lambda + 36$$

or,

$$44 = \frac{5l_1}{2} + \frac{l_2}{2}$$

$$14 = \frac{l_1}{2} + l_2,$$

A solution to the above system of equations is $l_1 = 16.44$ and $l_2 = 5.77$. 

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