Alpha-Rooting Method of Gray-scale Image Enhancement in The Quaternion Frequency Domain

Artyom M. Grigoryan and Sos S. Agaian

February 2017

Department of Electrical and Computer Engineering
The University of Texas at San Antonio
One UTSA Circle, San Antonio, TX 78249-0669, USA
Tel/Fax: (210) 458-7518/5589
amgrigoryan@utsa.edu sos.agaiian@utsa.edu
Outline

- Introduction
- New approach to image enhancement
- Gary-scale to quaternion models
- Measures of image enhancement
- 2-D Right-Side Quaternion DFT
- Enhancement of images in grays
- Examples
- Summary
Introduction

- New gray-scale image representations in the 4-D quaternion space allows for images to be processed as quaternion and color images in the frequency domain, by using the concepts of the 2-D quaternion unitary transforms, such as quaternion discrete Fourier transforms, wavelet, and others.

- In this paper, we use the $a$-rooting method of image enhancement by the 2-D discrete quaternion Fourier transform (DQFT).

- Preliminary results show: that the application of new models of gray-scale images in the quaternion space results in high quality gray-scale images, and can be effectively used for enhancing images.
**Block-diagram of Gray-scale Imaging**

- **Fig. 1** The block-diagram of gray-scale image processing in the frequency domain.
Diagram: Main Steps

1. Gray scale image is transformed into the quaternion image of size smaller than the original image,
   \[ \{f_{n,m}; \ n = 0 : (N - 1), \ m = 0 : (M - 1)\} \rightarrow \{q_{n,m}; \ n = 0 : (N_1 - 1), \ m = 0 : (M_1 - 1)\}, \]

   where
   \[ q_{n,m} = a_{n,m} + (i(q_i)_{n,m} + j(q_j)_{n,m} + k(q_k)_{n,m}) \quad i^2 = j^2 = k^2 = -1. \]

2. The quaternion image is processed in the frequency domain
   \[ q_{n,m} \rightarrow \hat{q}_{n,m}, \quad n = 0 : (N_1 - 1), \ m = 0 : (M_1 - 1) \]

   by using for instance, the concept of the 2-D QDFT.

3. The processed quaternion image is transformed back into gray-scales
   \[ \{\hat{q}_{n,m}; \ n = 0 : (N_1 - 1), \ m = 0 : (M_1 - 1)\} \rightarrow \{f_{n,m}; \ n = 0 : (N - 1), \ m = 0 : (M - 1)\} \]
Quaternion Numbers

The quaternion can be considered as a 4-D generation of a complex number with one real part and three imaginary parts

\[ Q = a + bi + cj + dk = a + (bi + cj + dk), \]

with three imaginary units \( i, j, \) and \( k. \)

\[ ij = -ji = k, \quad jk = -kj = i, \]
\[ ki = -ik = -j, \quad i^2 = j^2 = k^2 = ijk = -1. \]

Quaternion conjugate and modulus are

\[ \bar{Q} = a - (bi + cj + dk), \quad |Q| = \sqrt{a^2 + b^2 + c^2 + d^2}. \]
Gray-Scale-To-Quaternion Image Model

**Model**: Image $f_{n,m}$ of size $N \times M$ can be divided by 4 parts:

$$
\begin{array}{cccc}
. & . & . & . \\
. & f_{2n,2m} & f_{2n,2m+1} & f_{2n,2m+2} & f_{2n,2m+3} & . \\
. & f_{2n+1,2m} & f_{2n+1,2m+1} & f_{2n+1,2m+2} & f_{2n+1,2m+3} & . \\
. & f_{2n+2,2m} & f_{2n+2,2m+1} & f_{2n+2,2m+2} & f_{2n+2,2m+3} & . \\
. & f_{2n+3,2m} & f_{2n+3,2m+1} & f_{2n+3,2m+2} & f_{2n+3,2m+3} & . \\
. & . & . & . & . & .
\end{array}
$$

Let $f_e = \{(f_e)_{n,m}\} = \{f_{2n,2m}\}$, $f_i = \{(f_i)_{n,m}\} = \{f_{2n,2m+1}\}$, $f_j = \{(f_j)_{n,m}\} = \{f_{2n+1,2m}\}$, $f_k = \{(f_k)_{n,m}\} = \{f_{2n+1,2m+1}\}$, 

where $n = 0 : (N/2 - 1)$, $m = 0 : (M/2 - 1)$.

These four parts can be considered as components of the quaternion matrix which we call the quaternion image and denote by $q(f)$,

$$q(f) = (f_e, f_i, f_j, f_k) = f_e + (i f_i + j f_j + k f_k)$$
Gray-Scale-To-Quaternion Image Model

When transforming the gray-scale image into the quaternion subspace, we may assume that the components $f_i$, $f_j$, and $f_k$ are the set-components of the imaginary part of the quaternion image, as shown in this table.

| . | $E: f_{2n,2m}$ | $R: f_{2n,2m+1}$ | $E: 2n,2m+2$ | $R: f_{2n,2m+3}$ | . |
| - | $G: f_{2n+1,2m}$ | $B: f_{2n+1,2m+1}$ | $G: f_{2n+1,2m+2}$ | $B: f_{2n+1,2m+3}$ | . |
| - | $E: f_{2n+2,2m}$ | $R: f_{2n+2,2m+1}$ | $E: f_{2n+2,2m+2}$ | $R: f_{2n+2,2m+3}$ | . |
| - | $G: f_{2n+3,2m}$ | $B: f_{2n+3,2m+1}$ | $G: f_{2n+3,2m+2}$ | $B: f_{2n+3,2m+3}$ | . |
| - | . | . | . | . | . |

**Table 1.** The gray-scale image mapping to the RGB color model:

The letter $E$ stands for the gray-levels, $R$ for the red color, $G$ for the green color, and $B$ for the blue color.

We assume that the component $f_i$ is set to the red color channel, and $f_j$ and $f_k$ to the green and blue channels, respectively, as shown in the table.
Gray-Scale-To-Quaternion Image Model: Example

The gray-scale “pentagon” image of size $1024 \times 1024$.

The gray-scale image and color image of size $512 \times 512$ each represent the original gray-scale “pentagon” image of size $1024 \times 1024$ in the quaternion space.
Quaternion-TRANSFORM-BASED IMAGE ENHANCEMENT

The basic idea behind the frequency domain methods consists in computing a discrete unitary transform of the image, for instance the 2-D quaternion DFT (2-D QDFT), manipulating the transform coefficients by a operator $M$ of magnitude, and performing the inverse transform.

![Diagram](block_diagram.png)

**Fig. 4** Block-diagram of the quaternion Fourier transform-based image enhancement

The *a-rooting* method of image enhancement:

$$|Q(p, s)| \rightarrow M(|Q(p, s)|) = |Q(p, s)|^\alpha, \quad p, s = 0 : (N - 1).$$
Quat ernion Fourier Transform-Based \( \alpha \)-Rooting Image Enhancement Algorithm

Input is a quaternion image or color image and the value of the parameter \( \alpha \) is from the interval \((0,1]\).

- **Step 1.** Perform the 2-D QDFT of the color image.
- **Step 2.** Multiply the transform coefficients, \( Q_{p,s} \), by the quaternion factors \( C(p,s) = c|Q_{p,s}|^\alpha-1 \).
- **Step 3.** Perform the inverse 2-D QDFT.

The output is an enhanced quaternion image.
The imaginary part of the output is an enhanced color image.
Quantitative Measure of Image Enhancement: Gray-scale Image case

To measure the quality of gray-scale images and select “optimal” processing parameter (or parameters) for image enhancement, we consider the quantitative measure, which is known as EME – enhancement mesure estimation.

The quantitative measure of enhancement of the image processed by $\Phi$ transform $\{f_{n,m}\} \rightarrow \{\hat{f}_{n,m}\}$

$$EME_{a,\Phi}(\hat{f}) = EME_{L_1,L_2; a,\Phi}(\hat{f}) = \frac{1}{k_1 k_2} \sum_{k=1}^{k_1} \sum_{l=1}^{k_2} 20 \log_{10} \left[ \frac{\max_{k,l}(\hat{f})}{\min_{k,l}(\hat{f})} \right]$$

The image $f_{n,m}$ of size $N_1 \times N_2$ is divided by $k_1 k_2$ blocks of size $L_1 \times L_2$, for instance 7×7 or 9×9.
Alpha-rooting – 2-D DFT Image Enhancement: Example 1

The enhancement equals

\[ EME_{0.88}(\hat{f}) - EME(f) = 19.61 - 17.11 = 2.50 \]

**Fig. 2** (a) The original image, (b) the curve \( EME(\alpha) \), and (c) the image enhanced by the \( \alpha \)-rooting method.
Alpha-rooting by the 2-D DFT: Example 2

![Original Image](image1)

![Enhanced Image](image2)

(a) “pentagon” image 1024×1024, (b) the enhanced image, and (c) the curve of the enhancement measure calculated by 9×9 blocks.

**Fig. 3** Parameterized 2-D DFT image-enhancement
The images are in the RGB color space, \( f_{n,m} \rightarrow \hat{f}_{n,m} \)

\[
f = (f_R, f_G, f_B) \quad \text{and} \quad \hat{f} = (\hat{f}_R, \hat{f}_G, \hat{f}_B)
\]

EME for color images is calculated by

\[
EMEC(f) = \frac{1}{k_1k_2} \sum_{k=1}^{k_1} \sum_{l=1}^{k_2} 20 \log_{10} \left[ \frac{\max_{k,l}\{\hat{f}_R, \hat{f}_G, \hat{f}_B\}}{\min_{k,l}\{f_R, f_G, f_B\}} \right].
\]

Here, the maximum and minimum values of the image in the \((k,l)\)-th block are calculated as

\[
\max(\hat{f}) = \max(\hat{f}_R, \hat{f}_G, \hat{f}_B) \quad \text{and} \quad \min(\hat{f}) = \min(\hat{f}_R, \hat{f}_G, \hat{f}_B).
\]

For four-component quaternion images, the EMEC is calculated similarly (we can call it EMEQ).
Right-side 2-D Quaternion Discrete Fourier Transform

Let \( q_{n,m} \) be the quaternion discrete image of size \( N \times N \)
\( \mu \) be the pure unit quaternion, \( \mu^2 = -1 \).

The right-side 2-D QDFT is defined as

\[
Q_{p,s} = \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} q_{n,m} W_{\mu}^{np+ms}, \quad p, s = 0 : (N - 1).
\]

where

\[
W_{\mu} = W_{\mu;N} = \exp(-2\pi\mu/N)
\]

The inverse right-side 2-D QDFT is calculated by

\[
q_{n,m} = \frac{1}{N^2} \sum_{p=0}^{N-1} \sum_{s=0}^{N-1} Q_{p,s} W_{\mu}^{-(np+ms)}, \quad n, m = 0 : (N - 1).
\]
Right-side 2-D Quaternion Discrete Fourier Transform

Example: $512 \times 512$ quaternion “pentagon” image

$$Q_{p,s} = (Q_{p,s})_e + \left( i(Q_{p,s})_i + j(Q_{p,s})_j + k(Q_{p,s})_k \right), \quad p, s = 0 : 511$$

The $512 \times 512$ 2-D QDFT

Fig. 4 Magnitudes of (a) real component, $|Q_{p,s})_e|$, and imaginary component, $\sqrt{(Q_{p,s})_i^2 + (Q_{p,s})_j^2 + (Q_{p,s})_k^2}$, before (b) and after (c) cyclicly shifting to the center.
Alpha-rooting 2-D Quaternion DFT: Example 2 ...

Figure 5 shows the graph of the EMEC measure as a function of $\alpha$ with the same optimal value 0.89, for the same example with the quaternion “pentagon” image.

![Graph of EMEC measure](image)

**Fig. 5** The EMEQ of the alpha-rooting by the 2-D QDFT.

The curve of the enhancement measure EMEQ was calculated by $9 \times 9$ blocks.
Image enhancement: Examples

Fig. 6 (a) Enhancement functions for three channels of the tree image. (b,c) The $\alpha$-rooting by the 2-D QDFT.
Image enhancement: *Example*

![Fig. 7](a) The original 1024× 1024 gray-scale “pentagon” image and (b) the enhanced image after processing by the alpha-rooting in the quaternion space.
Image enhancement: Example

**Fig. 8** (a) The original image “5.3.02” and (b,c) the quaternion image.

**Fig. 9** (a) The original image and (b) the enhancement by $\alpha$-rooting in the quaternion space (by using the 2-D QDFT).
Fig. 10 (a) Four EME measures of the $\alpha$-rooting, and the imaginary parts of (b) the quaternion “pentagon” image, and the images enhanced by (c) the 0.92-rooting and (d) the 0.86-rooting.
Summary

• New approach of image enhancement is proposed for gray-scale images, which is based on the idea of transforming the image into the quaternion space, where the image can be enhanced and filtered by using the concept of the 2-D QDFT.

• The enhancement by alpha-rooting by 2-D QDFT is described.

• Preliminary results show that the application of the 2-D QDFT plus the alpha-rooting method can be effectively used for enhancing gray-scale images.

References

